Global Warming as a Manifestation of a Random Walk

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ABSTRACT

Global and hemispheric series of surface temperature anomalies are examined in an attempt to isolate any specific features of the structure of the series that might contribute to the global warming of about 0.5°C which has been observed over the past 100 years. It is found that there are no significant differences between the means of the positive and negative values of the changes in temperature from one year to the next; neither do the relative frequencies of the positive and negative values differ from the frequencies that would be expected by chance with a probability near 0.5. If the interannual changes are regarded as changes of unit magnitude and plotted in a Cartesian frame of reference with time measured along the x axis and yearly temperature differences along the y axis, the resulting path closely resembles the kind of random walk that occurs during a coin-tossing game.

We hypothesize that the global and hemispheric temperature series are the result of a Markov process. The climate system is subjected to various forms of random impulses. It is argued that the system fails to return to its former state after reacting to an impulse but tends to adjust to a new state of equilibrium as prescribed by the shock. This happens because a net positive feedback accompanies each shock and slightly alters the environmental state.

1. Introduction

The detection of when the prospect of an enhanced greenhouse effect global warming could be said to have hit public consciousness throughout the world is about as difficult to detect as that of the enhanced greenhouse effect itself. Perhaps the Villach (1980) Conference Statement could be taken as a useful starting point for general public awareness of the issue. In the ensuing decade of the 1980s, with the development of complex general circulation models, research and discussion about the subject have proceeded apace.

Arguments attempting to verify or, alternatively, to question an enhanced greenhouse effect have tended towards a two-pronged approach directed either to the predictions of the numerical models or to the analysis of time series of climatic measurements, particularly of temperature. We will concentrate here on the second method, confining our main attention towards hemispheric surface temperature observations.

One of the most comprehensive reviews of climatic change is that presented by Ellsaesser et al. (1986). The authors used various types of recorded data including precipitation, land glaciers, sea ice, snow cover, and sea level as well as surface, tropospheric, and stratospheric temperature records. The overall conclusions of this study were that the global mean surface–air temperature has increased, but whether the overall warming constitutes a climate change remains an unsolved problem. At about the time of this review Angell (1986, 1988) presented analyses of temperature trends in the troposphere for different heights and belts of latitude. Although trends were identified the relative magnitudes with respect to height and latitude did not agree with model predictions in every respect. Angell (1986) concluded that the data cannot be considered to provide conclusive evidence of anthropogenic influences on atmospheric temperature, while Angell (1988) concluded that in view of the various agreements and disagreements between the observational data and model predictions it was too early to state categorically that an enhanced greenhouse effect is already being observed. Karoly (1989) came to somewhat similar conclusions in finding that opposing temperature variations in the troposphere and stratosphere in the Northern Hemisphere are associated with many mechanisms and that the results of his analysis of upper air temperature observations could not be used to identify a specific mechanism. In contrast, Jones et al. (1986), using recently homogenized data over the land and oceans of both hemispheres during the past 130 years, found little trend in the 19th century, marked warming to 1940, relatively steady conditions to the mid-1970s, and subsequent rapid warming. They also noted that the warmest three years have all occurred in the 1980s, and that with regard to hypothesized

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warming due to increasing concentrations of CO_2 and other greenhouse gases, the overall change is in the right direction and of the correct magnitude. Ratcliffe (1990) remarks that 1989 was the 5th warmest year on record back to 1860, and that in this series six of the warmest ten years have been in the 1980s.

Wigley (1989) states that global warming is virtually certain and is likely to be substantial, perhaps as much as 3°C by the year 2050. The official publication of the Commission of the European Communities (Warwick et al. 1990) predicts a range of 0.5° to 2.5°C warmer than today by the year 2030. A best bet of 1° to 2°C is suggested as a reasonable assumption. The IPCC (Intergovernmental Panel on Climate Change) Report concludes that under a "business as usual" scenario, emissions of greenhouse gases will result in an increase of global mean temperature during the next century of about 0.3°C per decade, and this will cause a likely increase of about 1°C above the present value by 2025 and 3°C before the end of the next century. This increase should occur with an uncertainty range of 0.2° to 0.5°C per decade. Under other scenarios, which assume progressively increasing levels of control of greenhouse gas emissions, rates of increase of 0.2°C per decade and just above and below 0.1°C per decade are expected. On the basis of past records the IPCC Report judgement is that the global mean temperature has increased by 0.3° to 0.6°C over the last 100 years, with the five global-average warmest years being in the 1980s, and that this warming is consistent with climate variability. The report states that the unequivocable detection of the enhanced greenhouse effect from observations is not likely for a decade or more.

A recent study by Spencer and Christy (1990) looks at passive microwave radiometry from satellites to obtain more precise temperature information than has been available from the relatively sparse distribution of thermometers and radiosonde stations over the earth's surface. Analysis of the first 10 years of data (1979–1989) reveals a monthly precision of 0.01°C, with large temperature variability on scales from weeks to several years, but no obvious trend over the 10-year period.

a. *Random walk hypothesis*

The argument is put that the earth's climate system is subjected to periodic shocks consisting of random warming and cooling impulses. The impulses may be random in magnitude and frequency and may occur on time scales ranging from minutes to hundreds or thousands of years. On a time scale of days and weeks they may result from small variations in the solar flux or in large variations in cloud cover. Variations in cloud cover are unpredictable and arise in part from the genesis of cyclonic disturbances through baroclinic instability and from tropical storms, hurricanes, and typhoons produced through the operation of conditional instability. Such systems create temporary but vast cloud shields. Other longer term variations may accumulate within different time scales, i.e., biennial, ENSO, and solar cycle, not to mention the long-term periodicities due to the eccentricity of the earth's orbit, the tilt of the orbit, and the precession of the equinoxes. In this study we concentrate specifically on the variations within a time span of one year that accumulate and are observed as a mean global temperature expressed as an anomaly from a designated 30-year value. On this time scale the hypothesis states that the climate system loses at least part of its memory from one year to the next, i.e., the temperature does not revert to its previous value after it has been randomly pertured. The physical justification behind this argument is the fact that the climate system may within a year adapt to a slightly different equilibrium state as a consequence of positive feedback. Positive feedbacks such as changes in water vapour content, changes in sea and ice cover, and perhaps changes in cloud coverage are discussed by Mitchell (1989), who suggests that the cumulative feedback after a forced temperature change is positive.

b. *The arc-sine law*

Markov dependent coin-tossing is discussed by Darroch (1972). The specific condition that distinguishes the manner in which the year-to-year changes occur, from a non-Markov to a Markov random walk process involving two transition states, a positive or a negative change, is that the transition should not be determined by its previous state. Such a process is in contrast to processes where the past history of the system influences its future development. A random walk is a special case of a Markov process. The arc-sine law is a special case of a random walk and is applicable to coin-tossing experiments. Imagine such an experiment between two players, A and B. We may represent the progress of the game by a plot in a Cartesian frame of reference. Units along the x axis represent the number of trials. Units along the y axis represent gains or losses. Player A wins if the coin comes up heads and B wins if it comes up tails. The plot forms a random walk where each point indicates the cumulative gain or loss of A and B at any time specified by the value of N, the number of trials. We will adopt the convention that the area on the positive y side of the x axis, that is the area bounded by the plot and the x axis, represents time when player A is ahead, and the area on the negative y side of the x axis represents time when player A is behind. The state of play is then stated by the arc-sine law, contained in the expression:

$$P = \frac{1}{\pi}\int_0^{\alpha} \frac{dx}{[x(1-x)]^{1/2}} = \frac{2}{\pi}\sin^{-1}\alpha^{1/2}. \quad (1)$$

Equation (1) is derived from the transformation of a Riemann sum formulated from classical probability theory (Feller 1960). It states that in a coin-tossing

experiment (or in any other game involving two even chance probabilities, such as red and black in a game of roulette, or even and odd numbers in a game of bingo or keno), the probability that the coin will spend a proportion of the total game time (α) or less, assuming that the coin is tossed at equal intervals, on the positive or negative y side of a plot described is P. The curve for the expression is shown in Fig. 1. The interesting conclusion to be derived from this curve is that the fraction of time spent on the positive (or negative) y side of the x axis is much more likely to be nearly zero or 1 than to be the more intuitively expected fraction of 0.5. If we contract the y axis by doubling the scale on the ordinate of Fig. 1 we obtain the probability for the time less than α spent by the loser of the coin-tossing game on the positive side of the y axis. Equation (1) then becomes

$$P = \frac{4}{\pi} \sin^{-1}\alpha^{1/2}. \qquad (2)$$

It may be computed from Eq. (2) that it is ten times more likely for the loser to be ahead between 0% and 1% of the time than between 49% and 50% of the total game time. An example discussed by Feller (1960) is given in a somewhat unusual context. If two players toss a coin once a second for a whole year there is a 5% probability that the loser will be ahead for less that 13.5 hours and a 1% probability that the loser will be ahead for less that 32.4 minutes. Feller adds that few people would believe that a perfect coin would produce preposterous sequences in which no change of lead occurs for millions of trials in succession and yet this is just what a good coin will do rather regularly.

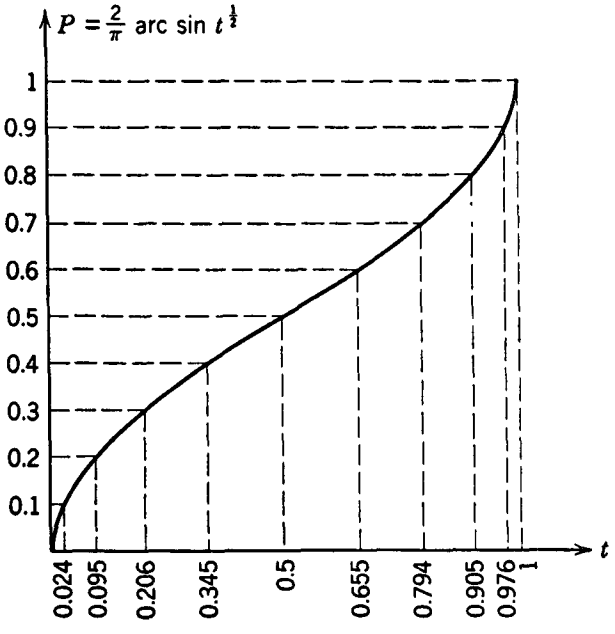

FIG. 1. The arc-sine law. The x axis marks the proportions of game time less than α that may be spent on the positive (winning) or negative (losing) y side of the x axis during a coin-tossing experiment. The y axis denotes the probabilities associated with the respective values of α (from Feller 1960).

Equation (2) is rigidly valid as N, the number of trials, approaches infinity, but Feller (1960) states that in practice the formula provides an excellent approximation even for values of N as small as 20, much less than the 130 observations of mean annual surface temperature anomalies available in the climatic record.

Figure 2 shows a random walk arising from plotting the results of a coin-tossing experiment that lasted for 10 000 trials. Note that in the upper part of the figure the x axis is marked in division of 100 trials each whereas in the lower two parts the x axis is marked in divisions of 100 trials each. Consequently, the upper section of the random walk appears more extended. It is noted that there are well-defined waves in the plots. Their wavelengths are approximately proportional to the square root of the number of observations. A first glance at the plot might convey the impression that the curve is similar to long period global temperature trends. If such a plot was of infinite length and one zoomed into a specific region of the plot one might regard the total perspective as an example of fractal geometry in chaos theory. Some specific features of Fig. 2 inherent in a breakdown of the results of the experiment into groups of 100 trials each are:

1) The most unequal distribution within any group of 100 trials was 39/61, $p = 0.027$.

2) After 10 000 trials heads were 21 down, but after 10 000 trials there is an even chance that the distribution will be ± 32 from 5000.

3) The greatest maximum gain in the game was heads 31 up after 4300 trials, $p = 0.34$.

4) The greatest increase occurred between trial 2100 and 4000, within 1900 trials when heads had gained 52 successes over tails, approximately $p = 0.02$. It is pertinent to mention that the reverse of this random walk is even more surprising. Only 70 steps out of the 10 000 are on the positive y side of the x axis. This fraction of 0.007 occurs with probability slightly greater than 0.1. The probability that the lead never changes in 10 000 trials is 0.0085.

An extension of the first arc-sine law already discussed is the second arc-sine law, which states that the maxima (or minima) are more likely to be found near the end (or beginning) of the series of values than anywhere else. More precisely, the probability of finding the first maximum (or minimum) at either $2k$ or $2k + 1$ time steps is the same probability that the plot spends $2k$ out of a total $2n$ time units on the positive y side of the x axis. The seemingly unusual fact that the probability distribution of time spent in the lead and the probability distribution of the position of the maxima are the same is no peculiarity of coin-tossing experiments (Feller 1960; Sparre Andersen 1953,

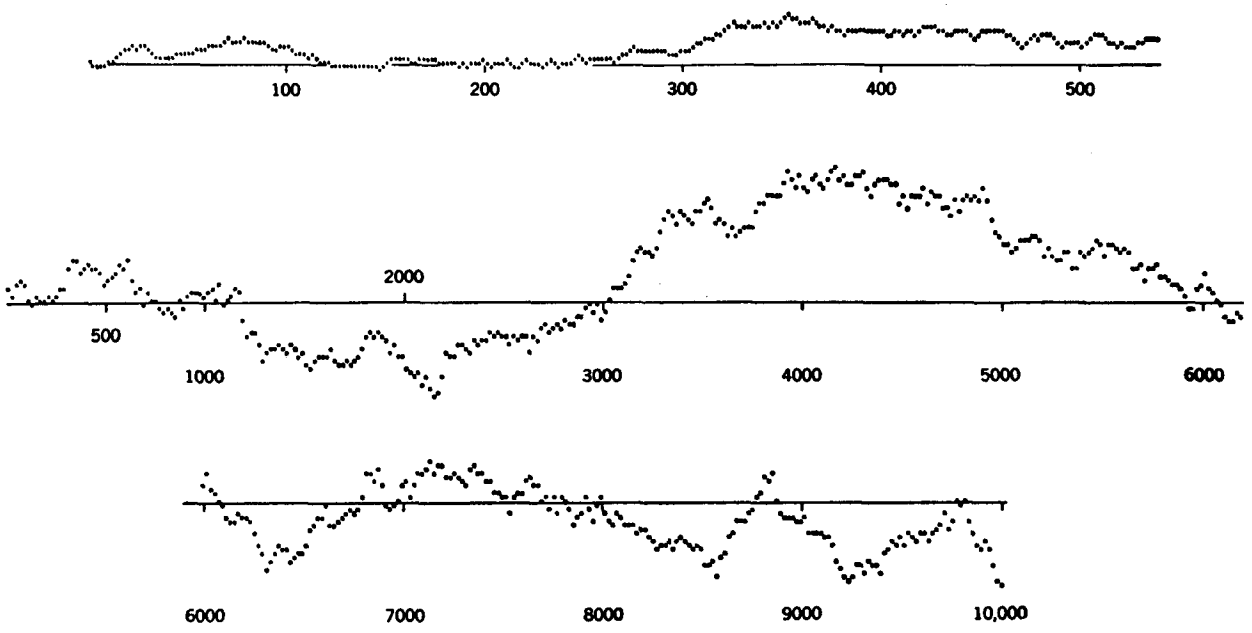

FIG. 2. The record of 10 000 trials of a coin-tossing experiment. In the upper section of the diagram the number of trials has been marked along the x axis in intervals of 100. In the two lower sections the number of trials has been marked at intervals of 1000. The wavelengths of the random walk tend to increase as the square root of the number of trials. The plot shows how many steps "heads" is ahead or behind "tails" (after Feller 1960).

1954). This result is of much importance when we look at current global temperature trends that appear to be reaching their highest peaks in the last decade. Far from being an unusual event, it is more probable to find a new peak at the end of the series than at any other time. The very existence of the peak within a current year draws attention to itself.

2. The global temperature series

The specific series of temperatures used in this study are the annual land and sea surface temperatures for the period 1861–1988, inclusive, combined into a single series and expressed as annual anomalies from the 1950–1979 mean (Jones 1988) for the two hemispheres. The series for the globe is the mean of the anomalies for the two hemispheres. The linear trends for the Northern and Southern hemispheres given by these series are 0.00378°C/yr and 0.00436°C/yr, and after detrending autocorrelation at lag 1 year is 0.4767 and 0.4063. The autocorrelations decay to negative at 12 years and 6 years, respectively. The series of anomalies as published were converted to a series of changes from one year to the next, that is, a series of the first difference of the anomalies. Counts were made of the number of individual warmings and coolings, and means and standard deviations were computed for each separately.

In looking at the results shown in Table 1 it should be kept in mind that the accuracy of the data is published to 0.01°C. In addition the overall precision of the data is open to some question when factors such as historical accuracy of instrumentation and unequal and sparse distribution of observations are taken into account. It is conceivable that the accuracy of any given annual value is only 0.1°C for the late 19th century and about 0.05°C for the post World War II averages (Jones 1990, personal communication).

Table 1 shows that the global means of the positive and negative values of the year-to-year changes of temperature are almost identical. This close agreement is obviously a fluke. Each annual value of the global series is the mean of the annual values for each hemisphere, so that the average values for the two hemispheres taken separately are slightly different from the global values calculated from the global series. In no case does the difference of the mean positive and mean negative interannual changes, either for the globe as a whole or within each hemisphere, reach the 10% level of significance by the two-tail test (Table 2). The absolute value of the mean of the coolings is greater than the mean of the warmings in the Northern Hemisphere, whereas the reverse is true in the Southern Hemisphere.

Table 2 shows the levels of significance for the differences of the cross-hemispheric means. The significant differences are those between Northern Hemisphere positive and Southern Hemisphere negative values, greater than 10% but not exceeding 5%, and between Northern Hemisphere negative and Southern Hemisphere negative values, which just exceeds the 5% level. The physical significance of the latter differ-

TABLE 1. Counts, mean values, and standard deviations of the positive and negative values of the changes in surface temperature from one year to the next for the 1861–1988 series of surface temperature anomalies.

	Number of warming years	Number of cooling years	Number of no change years	Mean positive °C	Mean negative °C	Standard deviation positive °C	Standard deviation negative °C
Globe	64 (67)	57 (60)	6	0.1027	0.1026	0.0704	0.0790
NH	63 (68.5)	53 (58.5)	11	0.1217	0.1336	0.0960	0.1045
SH	58 (61)	63 (66)	6	0.1164	0.0936	0.0801	0.0873
$\frac{NH + SH}{2}$	60.5 (64.75)	58 (62.25)	8.5	0.1192	0.1119	0.0880	0.0960

The numbers in parentheses are the counts when the no change values are evenly distributed between the positive and negative values. Even chance distribution is 67/60. The 5% significance level is 75/52 and the 1% significance level is 78/49.

ences probably relies on the fact that the Northern Hemisphere consists of approximately 0.4 land and 0.6 ocean, compared to 0.2 land and 0.8 ocean for the southern hemisphere. A larger land surface would most certainly be likely to cause greater interannual variability. A reason why the additional land surface in the Northern Hemisphere appears to give rise to larger mean coolings than warmings from year-to-year could be the result of radiative cooling over large areas of continental snow cover. The subsequent recovery would involve melting of snow and ice and warming of a wet land surface, which could be a slower process than the sudden occurrence of a widespread and intense snowstorm.

Having established that there is no significant difference between the means of the positive and negative values of the year-to-year changes for the globe or within the two hemispheres, it remains to look at the counts of the positive and negative values within the whole distribution and to compare these frequencies with those likely to occur by chance. The actual counts for the total of 127 cases are given in Table 1, while the expected frequencies are given in the caption to the table. The spread of observed frequencies is well within the probabilities to be expected by chance within the 10% level of significance. However, there is a tendency for the changes from one year to the next to be more frequent from one sign to the opposite sign than from one sign to the same sign. This result does not reach the 10% level of significance for either hemisphere and may be attributed to the effect of the quasi-biennial oscillation (QBO). The QBO has been studied by Angell and Korshover (1962), Brier (1978), and Quiroz (1981). A biennial tendency has been reported in the El Niño–Southern Oscillation (ENSO) phenomenon (Rasmusson et al. 1990). In the long run the effects of the QBO signal tend to cancel out, but the amplitudes and phases of the periodicities differ from one oscillation to the next and these differences themselves inject randomness into the observed interannual changes.

TABLE 2. Significance levels for the differences in the means of the positive and negative values of the year-to-year changes of surface temperature between the two hemispheres.

	NH +	NH −	SH +	SH −
NH +	—	n.s.	n.s.	10%
NH −	—	—	n.s.	5%
SH +	—	—	—	n.s.

3. Similarity between temperature changes series and random walks

a. *The recorded climatic series*

Figures 3(a–c) illustrate plots of year-to-year changes derived from the surface temperature records for the period 1861–1988. Each change is shown by a unit step up or down the y axis with time measured in years along the x axis. The changes are cumulative and the paths may be likened to the random walks produced by the coin-tossing game. Figure 3a shows the plot for the globe and figs. 3b and 3c for the Northern and Southern hemispheres. The plots show that the overall increase in temperature for the global series is 7 steps; the increase for the Northern Hemisphere series is 10 steps, while in the Southern Hemisphere there is a decrease of 5 steps. Figure 3d shows the plot for a series of randomly drawn odd and even numbers for a randomly chosen casino gambling game. The final trial shows odd numbers 11 ahead after 150 trials. The proportion of time spent on the negative y side of the x axis is shown by α and the probabilities of these proportions occurring by p, calculated from the arc-sine equation. These values of α all have probabilities within a range to be expected in a random walk. The temperature plots of real data and the casino game plot also show evidence of maxima appearing towards the end of the run, as might be expected from arc-sine law predictions. The latter result is a most interesting one in view of the fact that six out of the past ten years have been the warmest in the past 100 years or longer. Yet, if the record portrayed a random walk, such a

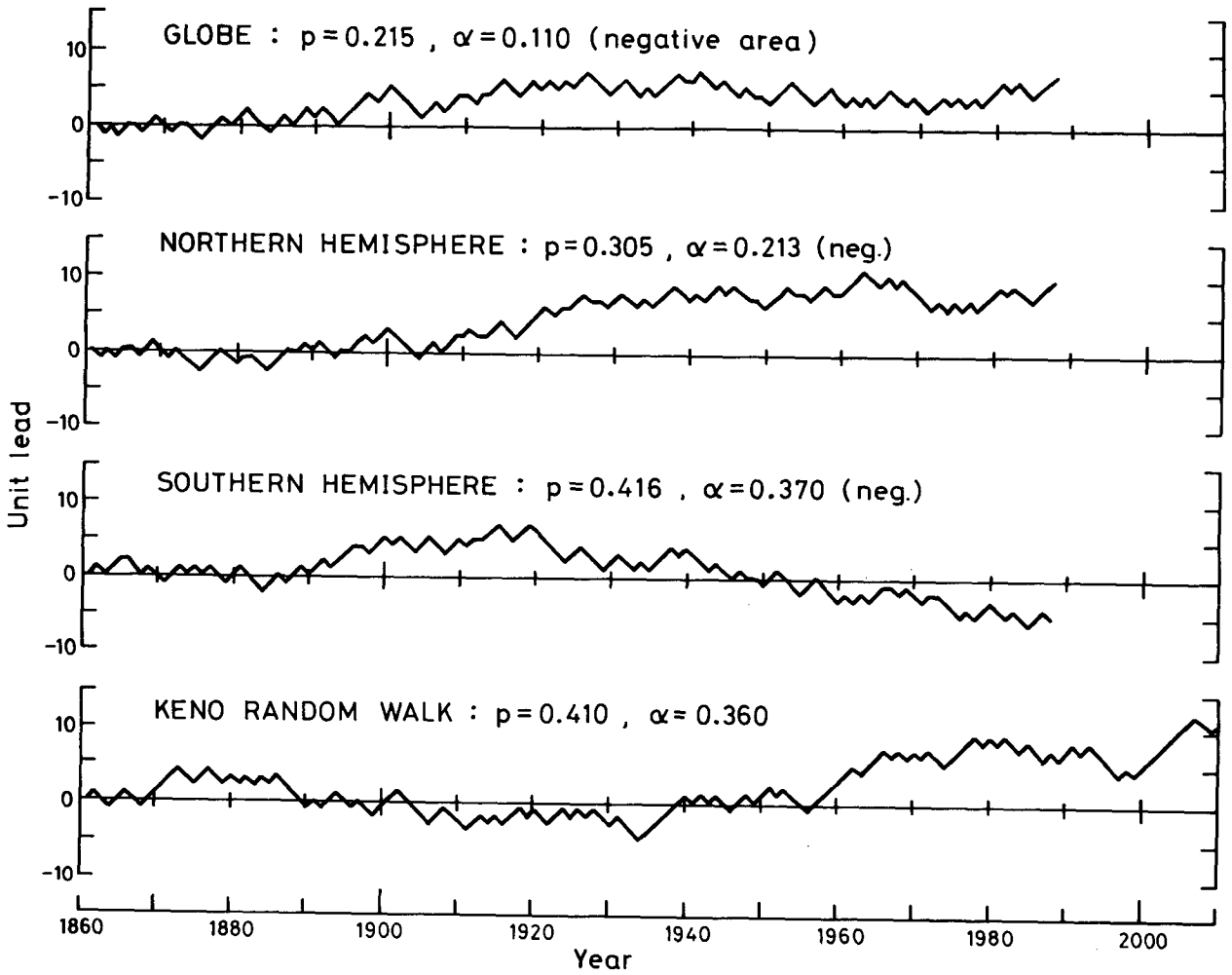

FIG. 3. Plots of the changes in temperature from one year to the next from the 1861–1988 series of mean surface temperature anomalies. Each change has been given unit magnitude. Values of the proportion of time that α spent on the negative y side of the x axis have been calculated from the arc-sine law together with the probabilities associated with those values. The upper plot is from the global series, and the middle two plots are for the two hemispheres, as stated. The probabilities are all within ranges likely to be expected if the plots constituted random walks. The lower plot is a true random walk derived from the sequence of odd and even numbers in a casino gambling game.

result would be the most likely one. That is not to say that the temperature record is a random walk, but that it does possess similar features.

b. Randomly generated series

The cases discussed so far have concerned coin-tossing games or temperature series where the changes from one year to the next have been of unit magnitude. In order to examine more realistic cases, series were constructed wherein the changes from year-to-year differed from those previously described. Instead of having transition probabilities of 0.5 up or down the y axis with unit magnitude, we will now consider transitions where the probabilities of the magnitudes of the transition up or down the y axis are determined by the normal probability distribution. A number of random normal distributions composed of 100 values each were generated. In generating the frequencies the perfect normal distribution was given a mean of zero and a standard deviation equal to that of the interannual changes obtained from the global series of surface temperature anomalies. Thus the perfect normal distribution simulated the interannual changes of temperature with zero trend. The actual frequencies produced by such random generation differed, of course, from the exact distribution and produced marked trends when the values were added cumulatively.

The operation of adding the sequential values of the randomly generated sets of values is justified by the positive feedback hypothesis already formulated. It is open to question whether or not in the real world the chaotic nature of incoming impulses could result in a non-Markov process as a result of which moderately

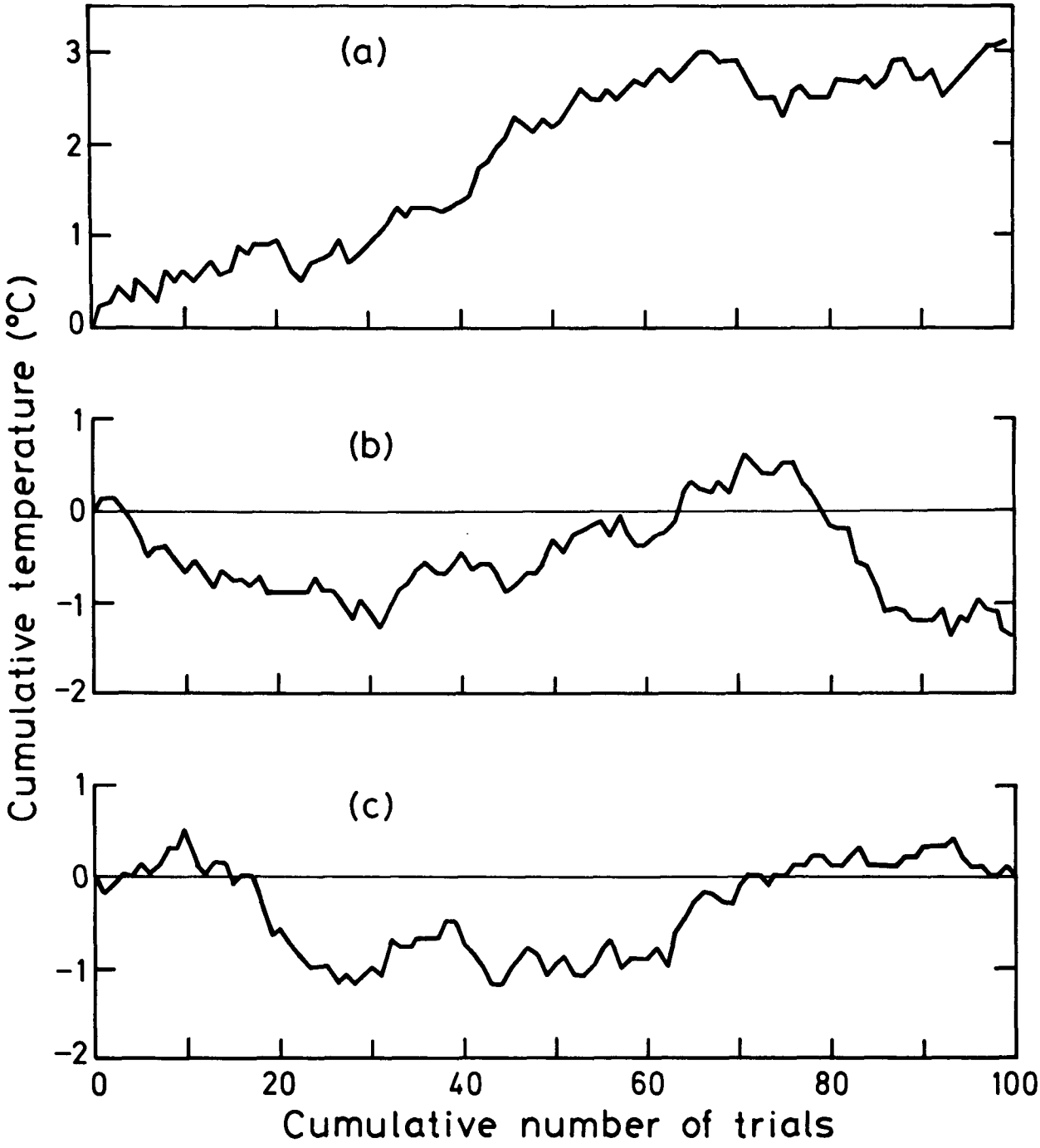

FIG. 4. Three plots of random walks derived from randomly generated normal frequency distributions. The sequential values in the order generated have been added cumulatively. The three plots were selected from a total of five distributions generated. A mean of zero and a standard deviation equal to that of the global series of the changes of temperature from one year to the next were used for the random generation of the normal distributions.

large trends over decades and centuries could occur. A model run (Hansen 1990, Fig. 1) shows a warming trend of 0.4°C over 50 years, followed by a cooling trend of 0.5°C over 15 years for a scenario that excludes variations in climate forcing.

Figures 4(a,b,c) illustrate three cases of random walks plotted from five random generated normal frequency distributions of 100 values each. A mean zero and standard deviation of 0.1238°C, drawn from the global mean annual temperature series of interannual

changes were used for each of the curves generated. The three curves shown were chosen from the five to illustrate that in such a small selection of randomly generated distributions, wide fluctuations of trends can occur. Figure 4(a) shows a case where the final temperature is 3.06°C higher than at the starting point. Figure 4(b) shows a case where the final temperature is 1.33°C lower than the starting point, while Fig. 4(c) shows a case where the final temperature returns to its starting point. In plotting the points, the cumulative values were rounded off to an accuracy of 0.1°C. In simulating analogues to the global temperature, the time interval between the individual points is considered to be one year. The trends for a hundred years in the first two plots exceed the actual global trends over the past 100 years (IPCC 1990) by factors of nearly seven and three, respectively. In Fig. 4(a) there is no return to the origin ($y = 0$) and maximum occurs at the last point. We remember from the arc-sine law that the probabilities of these two events are almost equal for the coin-tossing game. In that case, for 100 trials the probability that there is no return to the origin is about 0.08. (Feller 1960, theorem 2, p. 83). In cases shown in Fig. 4 we have random variables with probabilities determined by the normal distribution instead of one variable with a probability of 0.5. In Fig. 4(b) the curve crosses the axis three times, the final point is a minimum. In Fig. 4(c) the path also crosses the axis three times. In this case the overall trend is zero.

4. Conclusions

It is generally accepted that there has been a global warming of surface temperature of about 0.5°C over the past century. The manner in which this increase in temperature has occurred is examined. This has been done by reconstituting the series of global mean surface temperature anomalies into a series of changes of temperature from one year to the next. This series constitutes the interannual variability of the global climate system. If there has been an overall positive trend in the temperature this must result either from the fact that the mean of the positive values of the changes from year-to-year are greater than the mean of the negative values, or from the fact that there are more positive values than negative values, or that both of these conditions are met. An analysis of 128 years of anomalies of surface temperature of the globe and of the separate hemispheres shows that there is no significant difference between the means of the positive and negative values, and furthermore that the frequencies of the positive and negative values are those to be expected by chance with a probability not far removed from 0.5. Thus, both of the observed properties of the positive and negative values that give rise to a trend can be attributed to chance and on this evidence no conclusion can be drawn as to whether they have been forced by some physical means such as a continuous increase in greenhouse gases. In the context of these results it is suggested that a plot of the interannual changes resembles a random walk, the characteristics of which are governed by the arc-sine law. The first important property of the arc-sine law is that a parcel undergoing a symmetric random walk is much more likely to be in the lead almost all the time or hardly ever, rather than about half of the time. The second property of the law is that the first maximum (or minimum) is more likely to occur near the end or the beginning of the walk than in the middle. Both these features are exhibited by plots of the series of interannual changes in which each change has unit magnitude. Series of interannual temperature changes are then simulated. The latter are drawn from randomly generated normal frequency distributions with a mean of zero and a standard deviation equal to that of the global series of temperature changes. Such a perfect normal distribution would, of course, show no trend. The sequential individual values so generated are then added cumulatively and plots drawn for several cases of 100 values each. The plots show trends considerably greater than those observed in the real world dataset of surface temperatures. They also exhibit the properties of the arc-sine law. It is concluded that the global series could have arisen from random fluctuations and could therefore be analogous to arc-sine law governed random walks.

A physical hypothesis is put forward that supports the analogue between the interannual variability and a random walk. It is that the climate system loses a part of its memory as a consequence of positive feedbacks after interannual fluctuations on the time scale of a year. At the same time the possibility of a non-Markov type process is not ruled out.

It is important to examine all ways and means by which the observed data series develop trends before facing hard and fast conclusions that any particular activity is the one and only responsible agent.

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