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Chaos, fractals and self-organization in coastal geomorphology: simulating dune landscapes in vegetated environments

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Abstract

Complex nonlinear dynamic systems are ubiquitous in the landscapes and phenomena studied by earth sciences in general and by geomorphology in particular. Concepts of chaos, fractals and self-organization, originating from research in nonlinear dynamics, have proven to be powerful approaches to understanding and modeling the evolution and characteristics of a wide variety of landscapes and bedforms. This paper presents a brief survey of the fundamental ideas and terminology underlying these types of investigations, covering such concepts as strange attractors, fractal dimensions and self-organized criticality. Their application in many areas of geomorphological research is subsequently reviewed, in river network modeling and in surface analysis amongst others, followed by more detailed descriptions of the use of chaos theory, fractals and self-organization in coastal geomorphology in particular. These include self-organized behavior of beach morphology, the fractal nature of ocean surface gravity waves, the self-organization of beach cusps and simulation models of ripples and dune patterns. This paper further presents a substantial extension of existing dune landscape simulation models by incorporating vegetation in the algorithm, enabling more realistic investigations into the self-organization of coastal dune systems. Interactions between vegetation and the sand transport process in the model—such as the modification of erosion and deposition rules and the growth response of vegetation to burial and erosion—introduce additional nonlinear feedback mechanisms that affect the course of self-organization of the simulated landscape. Exploratory modeling efforts show tantalizing results of how vegetation dynamics have a decisive impact on the emerging morphology and—conversely—how the developing landscape affects vegetation patterns. Extended interpretation of the modeling results in terms of attractors is hampered, however, by want of suitable state variables for characterizing vegetated landscapes, with respect to both morphology and vegetation patterns.

Keywords: Chaos; Fractals; Self-organization; Coastal dune; Simulation; Vegetation

1. Introduction

Research in nonlinear dynamic systems has grown rich and varied as notions of chaos, fractals and self-organization have been recognized in virtually all physical and human sciences, ranging from economics and linguistics to physics and geomorphology. This paper reviews the principal applications of these concepts in geomorphology, particularly in coastal geomorphology, and presents an exemplary self-organization model for the simulation of aeolian dune landscapes in vegetated environments. Although this paper is not intended as a rigorous and comprehensive review of chaos, fractals and self-organization in general, a brief overview of the basic ideas and terms involved is appropriate in order to appreciate their application in geomorphology. For comprehensive examination of these concepts, the reader is referred

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to Gleick (1987) and Kauffman (1995) for popular descriptions, while the more rigorous mathematical underpinning and specific algorithms may be found in Turcotte (1992) and Strogatz (1994).

2. General concepts

2.1. Chaos theory

Chaos theory is epitomized by the so-called 'butterfly effect' detailed by Lorenz (1963). Attempting to simulate numerically a global weather system, Lorenz discovered that minute changes in initial conditions steered subsequent simulations towards radically different final states. This sensitive dependence on initial conditions is generally exhibited by systems containing multiple elements with nonlinear interactions, particularly when the system is forced and dissipative. A system is said to be forced when its internal dynamics are driven by externally supplied energy (e.g. solar energy driving the global weather system), and a system is considered dissipative when 'useful' energy—in terms of its ability to perform work—is converted into a less useful form, most prominently through friction (also referred to as damping).

Sensitive dependence on initial conditions is not only observed in complex systems, but even in the simplest logistic equation model in population biology (May, 1976). This recursive equation describes the size of a self-reproducing population, P, at time $t+1$ as a nonlinear function of the population at time t: $P_{t+1}=rP_t(1-P_t)$. The value of the parameter r determines whether a population stabilizes at a constant size, oscillates between a limited sequence of sizes, or whether the population size behaves chaotically in an unpredictable pattern. In the latter case, a minute variation in the value of r results in a dramatically different population size after a specific period of time. Graphing the population size reached after a fixed number of iterations (P_T) against the parameter r generates a bifurcation diagram (Fig. 1), which

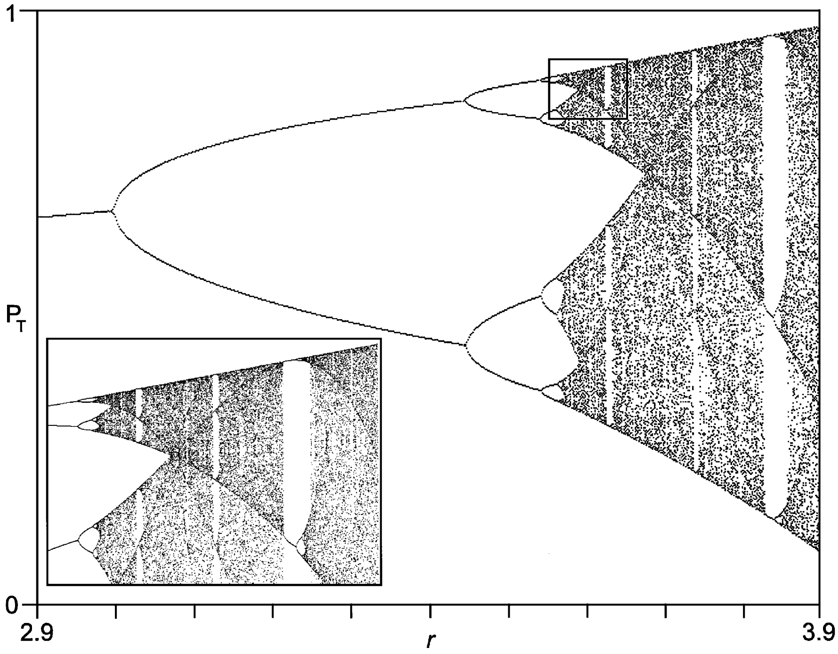

Fig. 1. The standard depiction of the bifurcation diagram for the logistic equation. Each dot charts the population size after 1000 iterations, P_T, of the recursive equation: $P_{t+1}=rP_t(1-P_t)$ for a value of r. The initiating value of P, P_0 is set to 0.66. Upon magnification of an apparently chaotic region (boxed) self-similar areas of order appear (inset). Magnification in this sense means a refinement of the precision of the values of r. Both this figure and Fig. 2 were created using the computer program "Fractint" developed by The Stone Soup Group (freeware).

contains regions with singular equilibrium populations for low values of r, bifurcating into an oscillating population as the parameter r increases and in turn deteriorating into a chaotic pattern as r reaches a critical value (Strogatz, 1994, p. 353). Within the chaotic regions, however, smaller areas of stable periodicity are discernible (associated with certain minute ranges of the value of r), and these stable areas appear over and over on every possible scale of examination. The self-similarity or repetition of this pattern across different scales is identified as a fractal.

2.2. Fractals

Fractals are defined as geometric objects that are self-similar under a change of scale, i.e. their shape remains the same under any magnification or reduction. The object consists of elements of a certain dimension embedded in a higher dimensional space, e.g. a distribution of points (dimension zero) on a line (one-dimensional), or a collection of planes (two-dimensional) embedded in a three-dimensional Euclidean space. A fundamental property of fractals is their 'fractal (also fractional) dimension', a noninteger value between the dimension of the constituting elements and the embedding dimension. Because of the self-similar pattern extending over an infinite range of scales, a fractal curve (composed of one-dimensional line elements) embedded in a two-dimensional plane, for example, has an infinite length within a finite area of the plane. The fractal dimension characterizes the extent to which the fractal 'fills up' the embedding space and, in this example, will attain a value between 1 and 2.

2.3. Attractors

The evolution of a dynamic system through time can be observed by tracing the instantaneous values of n state variables in n-dimensional space, the phase-space. A system in a steady state will appear as a point in phase-space, while a stable oscillator traces a closed loop through phase-space. The point and the closed loop are both attractors for their respective systems, i.e. the systems develop toward those states regardless of a range of boundary conditions and perturbations. A forced and damped oscillator (such as a magnetically driven pendulum with friction, for example) may be represented in a 2D phase-space by its instantaneous angular deflection and speed (the two state variables). An over-damped oscillator will spiral towards a point-attractor as it grinds to a halt, while under a range of forcing–damping ratios, the oscillating system will trace out a closed loop in phase-space. However, this nonlinear system also exhibits chaotic behavior under the right conditions, and it traces as a fractal in phase-space. This fractal is the attractor for the system in phase-space, termed a 'strange attractor'.

Fractals can thus be temporal, spatial or phase-space manifestations of chaos in nonlinear dynamic systems. Fractals are recognized in the time series of cotton prices and stocks and options (Mandelbrot, 1963), and in the occurrence of noise in communication lines (Berger and Mandelbrot, 1963). Spatial fractal patterns are recognized in a multitude of natural elements—at least over a certain range of scales—such as coastlines, snow flakes, fern leaves and the human lung. Fractals in phase-space can either be attractors themselves, i.e. strange attractors, such as the bifurcation diagram of the logistic equation, or they can constitute the dividing line between separate attractor basins in phase-space (Fig. 2).

2.4. Self-organization

The scale-invariance of fractals is frequently coupled with self-organization in nonlinear dynamic systems consisting of large aggregations of interacting elements. As such a system moves on a strange attractor in phase-space, any particular length scale from external forcing is lost ('forgotten') and instead, the smallest length scale of the individual elements propagates its effect across all scales. This generates pattern formation that may or may not exhibit fractal properties. An example is Rayleigh–Benard convection, where individual fluid motions are chaotic while a pattern of convection cells is formed that scales with the viscosity of the fluid (instead of with the amount of heat dissipation or the dimensions of the container). The emergence of structure, or order, in a system through its internal dynamics and feedback mechanisms is the essence of self-organization, as opposed to the generation of regularity as a result of external forcing. In a thermodynamic perspective, self-organization arises in nonlinear systems that are far from equilibrium and dissipative (irreversible). Coherent motions and pat-

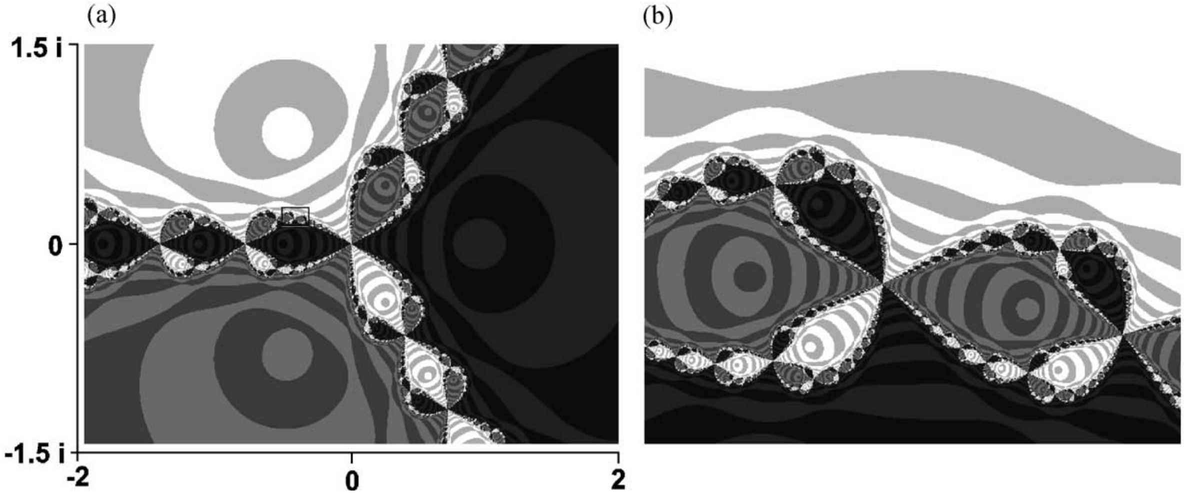

Fig. 2. (a) Fractal boundaries between three attractor basins in the Argand plane (whose axes are defined by the real and the imaginary number lines). The image shows how Newton's method for solving equations leads from any starting point in the plane to one of the three complex roots of the equation $x^3-1=0$. Newton's method starts with an initial guess, x_0, which is improved upon according to the iterative algorithm: $x_{n+1}=(2x_n^3+1)(3x_n^{-2})$. The gray tones indicate the attractor basins and the striping reveals the attractors, i.e. complex roots at: $x=1$, $x=-½+½\sqrt{3}\,i$ and $x=-½-½\sqrt{3}\,i$ at the center of the concentric bands. (b) Magnification of the boxed area in (a) reveals the fractal nature of the boundary between the attractor basins.

terns created in such systems are therefore called dissipative structures (Kauffman, 1995). Further thermodynamic interpretations of complex systems lead to principles of minimum entropy production in open systems and maximum entropy states in closed systems (Prigogine and Stengers, 1984). The stochastic interpretation of these entropy principles in complex systems can in turn be related to information theory (Shannon and Weaver, 1949; Jaynes, 1957; Brillouin, 1962; Kapur and Kesavan, 1992).

2.5. Self-organized criticality

A variation on the self-organization concept is the model of self-organized criticality proposed by Bak (1996). The prototypical example is the accumulating sandpile, in which the nonlinear dynamics between disturbed and avalanching sand grains retain the system in a critical state with the slopes of the pile at the angle of repose (Bak et al., 1988). The properties (such as angularity and size of the sediment) of the smallest element—the grain—determine the large-scale properties of the system as a whole (the critical angle of repose). A small disturbance (e.g. the addition of another grain of sand at the top) can trigger avalanches that, constrained only by the size of the pile itself. Power laws, such as the Gutenberg–Richter law of earthquake frequency and magnitude, are manifestations of self-organized criticality (Bak, 1996). Furthermore, the temporal signatures of such systems display a characteristic self-similar (fractal) response, where earthquakes or avalanches, for example, occur over all possible time scales. This behavior is manifested in a $1/f$ power spectrum (i.e. the power is inversely proportional to the frequency), as opposed to the uniform power spectrum of a purely stochastic process (Bak et al., 1987).

On a final note, it must be remarked that some of these ideas are controversial, especially with regards to their application to real physical systems. Furthermore, it is generally acknowledged that the peak of popular research interest into chaos, fractals and self-organization has passed. At the same time, it is also recognized, however, that these concepts still provide valuable new insights and approaches and offer certain distinct advantages when dealing with systems containing large collections of elements involving multiple or complex interactions. As such, they have been adopted as models and analysis tools in a broad range of scientific disciplines.

3. Geomorphology

Geomorphology presents an obvious arena for the application of chaos theory, fractals and self-organization concepts, as nearly all landscapes exhibit a range of nonlinear dynamic interactions between system elements. Further, many features of the natural landscape have a fractal-like appearance. Indeed, one of conceptions of fractals evolved through an analysis of the set of measurements of Richardson (1961) of Britain's coastline by Mandelbrot (1967), who has subsequently described a host of natural features in terms of fractals (Mandelbrot, 1982). Fractal dimensions are used in geomorphology primarily as a means of descriptive parameterization of patterns and landscape topography, e.g. as a measure of the roughness of a surface. Fractal dimensions can be determined two-dimensionally from contour lines or cross-sectional profiles, or in three dimensions using DEMs (Burrough, 1981; Turcotte, 1992; Tate, 1998). An extensive review of methods of fractal measurement of landscapes is found in Xu et al. (1993). Though landscape surfaces are not self-similar ad infinitum, it is found that fractal dimensions do capture some aspect of the surface roughness over a limited range of scales that other morphometric measures do not (Klinkenberg, 1992; Outcalt et al., 1994). However, comparison of fractal dimensions obtained from different methods can be problematic and error estimates are difficult to determine (Andrle, 1992; Xu et al., 1993). Thus, Andrle (1996) has shown that the coastline of West Britain does not conform to one single fractal dimension, contrary to Mandelbrot's earlier findings. While fractals and power-law distributions are widely applied for the analysis of landscape surfaces, fractional dimensions are also used as a differentiating characteristic of granular material in terms of fragmentation (Turcotte, 1986) and grain shape (Orford and Whalley, 1983; Kennedy and Lin, 1992). In soil hydraulics, an extensive body of literature exists dedicated to the use of fractal descriptions in water percolation problems, rock fracture, soil aggregates and ground water flow (Neuman, 1990; Anderson et al., 1998; Marrett et al., 1999).

In fluvial geomorphology, an entire field has developed around the analysis, interpretation and modeling of river networks in terms of fractals and self-organization. In particular, it has been found that well-established characteristic properties of river networks, such as Horton's power laws of bifurcation and stream-order length (Horton, 1945) and the power law of length and basin area of Hack (1957), are indicative of a fractal (self-similar) stream pattern (Rinaldo et al., 1993; Claps et al., 1996). These properties can be explained as resulting from self-organized criticality in the development of incised (erosional) landscapes, where the critical state is one of minimum energy dissipation (i.e. minimum entropy production) (Rinaldo et al., 1993; Rigon et al., 1994; Rodriguez-Iturbe and Rinaldo, 1997), harkening back to the random-walk models and entropy concepts of Leopold and Langbein (1962). Specifically, Rodriguez-Iturbe et al. have developed numerical models that simulate the development of river networks on cellular grids based on a few simple hydraulic sediment transport relations. In this algorithm, erosion (and subsequent basin development) only occurs when local shear stress—derived from local slope and catchment area—exceeds a critical threshold. This threshold-dependent feedback mechanism generates a fractal river network whose global energy dissipation is at a minimum (measured as changes in potential energy resulting from down slope mass-transport) and which displays the same power-law characteristics (e.g. Horton's and Hack's; see above) found in natural river networks (Rodriguez-Iturbe and Rinaldo, 1997, pp. 379–393).

More recently, the concept of self-organized criticality has been applied to braided rivers (Murray and Paola, 1994; Sapozhnikov and Foufoula-Georgiou, 1999) and tidal channels (Cleveringa and Oost, 1999), while further fractal analysis and self-organization modeling of river networks have been performed by Talling (2000) and Veneziano and Niemann (2000). However, field evidence does not always seem to support the thesis of statistical self-similarity of river networks (Beauvais and Montgomery, 1997).

Concepts of self-organization and chaos are now commonly found in many areas of research in the earth sciences, e.g. in the distribution and development of soils (Culling, 1988; Phillips, 2000), patterned vegetation (Klausmeier, 1999), patterned peri-glacial ground (Werner and Hallet, 1993) and riffle–pool sequences in mountain streams (Clifford, 1993). A host of numerical models has been developed simulating the evolution of periodic bedforms as a result of self-organization (see below). A recent issue of the *Journal of Hydrology* was dedicated solely to chaos theory in hydrology (Sivakumar, 2000), and many general review articles have appeared over the past decade (Huggett, 1988; Hallet, 1990; Malanson et al., 1990, 1992; Phillips, 1994, 1995, 1999; Werner, 1999). Many of the latter papers stress the fact that although concepts of chaos and self-organization are clearly valuable for geomorphology, several important issues remain to be addressed. Firstly, prevalent noise in natural geomorphic systems often obscures any truly chaotic or fractal pattern or signal present. Secondly, most current applications are limited to idealized numerical modeling of systems, while the quantity and quality of field evidence for testing these models are mostly inadequate, especially considering the large numbers of data points that are required on relatively small spatial and temporal scales. Lastly, there exists a variety of definitions and interpretations of self-organization and chaos in the literature and the associated diversity of methodology for analyzing geomorphic systems (e.g. power laws, criticality, entropy maximization, entropy production minimization, strange attractors, fractal dimensions, etc.) can be confusing, hampers comparison and is often challenging for geomorphologists not well versed in quantitative methods.

4. Coastal geomorphology

Coastal systems can be categorized as nonlinear dissipative complex systems as wind and wave energy is dissipated in the coastal zone and the interactions between morphology, sediment transport and fluid dynamics are strongly nonlinear. Southgate and Beltran (1998) and Southgate and Moller (2000) investigated the response of beach morphology to hydrodynamic forcing on monthly to decadal time scales in terms of self-organized behavior. By means of fractal analysis of beach-level time series at various locations along the cross-shore profile, they discovered that different parts of the shore face exhibit different degrees of self-organized behavior. At Duck, North Carolina, most notably the dune and upper shoreface zones display a fractal response that is indicative of self-organized behavior, while the inner and outer bar zones present mostly random Gaussian time series. Southgate and Moller (2000) relate these differing response regions to the different degrees and temporal scales of hydrodynamic forcing. They argue that the morphodynamic response in the bar zones is forced by the mixture of breaking and nonbreaking waves, a Gaussian forcing, whereas the upper shoreface zone and the dune zone experience much less external forcing by waves (in the first case because waves are not breaking yet, in the second because most wave energy is dissipated already). In the latter zones, self-organization of the profile is, therefore, more predominant.

In oceanography, various researchers have recently investigated water levels, wave climates and ocean currents in terms of chaotic behavior and self-organization. Growing out of a large body of literature concerning chaos and self-organized coherent structures in turbulent fluid flows (cf. Takens, 1981; Debnath and Riahi, 1998), Seidov and Marushkevich (1992) investigated the development of large-scale ocean currents resulting from stochastic forcing through self-organization mechanisms. Other research has shown that deep- and shallow water gravity waves exhibit a fractal surface both spatially and in time evolution (Elgar and Mayer-Kress, 1989; Stiassnie et al., 1991). Ohta and Kimura (1996) analyzed time series of wave height at three Japanese ports for chaotic behavior and tried to use that information to predict significant wave height, with mixed results. Frison et al. (1999) applied chaos theory to the analysis of ocean water levels measured at different types of coastlines and showed that a chaotic characterization extracted from the time series (so-called 'Lyapunov exponents') can be used to distinguish different tide zones and water level variability.

Perhaps the most widely known application of self-organization concepts in coastal geomorphology is the beach cusp model by Werner and Fink (1993). In this 3D numerical simulation model, only the basic processes of swash flow and sediment transport are incorporated in a greatly simplified algorithm. The nonlinear element in this system is the sediment flux being proportional to the cube of the flow velocity. The forcing is the initial shoreward velocity given to the swash, while the dissipation in the system largely results from the leveling effect of the enforcement of the angle of repose (a friction term) through avalanching. The nonlinear feedback between altering beach morphology and swash flow dynamics (and hence, sediment transport) produces a regular pattern of cusps and horns out of an initially plain Gaussian topography. Werner and Fink relate the mean beach cusp spacing to the swash excursion length—mainly a function of the beach slope—and have noted the good agreement with field observations. While predictions of cusp spacing are in close agreement with those from the standing edge wave model by Guza and Inman (1975), this also means that it is difficult to differentiate between the two mechanisms in field tests. Since the introduction of the Werner and Fink model, several investigations have been conducted to assess its merits and test both the self-organization and the edge wave models against field evidence (Allen et al., 1996; Coco et al., 1999a,b). These studies have reinforced the notion that both models are equally viable, but that only elaborate and extensive field measurements of swash zone dynamics will be able to distinguish which of the two mechanisms (or a simultaneous presence) is acting in the creation of beach cusps.

Cellular automaton models similar to the beach cusp model have also been applied to the simulation of aeolian ripples and dunes. Anderson (1990) developed a 2D cellular simulation model based on the dynamics between saltation, reptation and ripple evolution established by Anderson et al. (Anderson, 1987; Anderson and Haff, 1988). This model is driven by the high-energy impact of saltating grains ejecting reptating grains that form small undulations. Nonlinear feedback interactions occur between the angle of impacting grains, the reptating grains (via a splash function) and the evolving surface and its local slopes. Through progressive coalescence and mergers of small undulations, ripples evolve at a particular dominant wavelength, dependent on the saltation impact angle. Although the external driving force in this system, the impacts of saltating grains, is entirely stochastic (saltating grains are introduced randomly on the grid), a distinct ripple pattern spontaneously emerges through self-organization by the internal nonlinear dynamics. Later refinements of the model have also been capable of simulating grain size segregation and stratigraphy in ripples (Anderson and Bunas, 1993). Several other authors have presented similar numerical models (Nishimori and Ouchi, 1993; Werner and Gillespie, 1993; Vandewalle and Galam, 1999), while various analytical models have shown the fundamental instability of a flat surface subject to saltation impacts and the inevitable development of ripples (McLean, 1990; Werner and Gillespie, 1993; Hoyle and Mehta, 1999; Prigozhin, 1999; Valance and Rioual, 1999).

Numerical simulation of sediment transport in terms of moving slabs of sediment has also proven to be very amenable to the modeling of aeolian dunes. Werner (1995) applied this approach, pioneered with the aforementioned beach cusp model, to aeolian sand transport and the development of various types of dune patterns. This 3D model can simulate different dune-forming conditions in terms of varying wind directions, sediment flux and sand supply. The algorithm consists of an elementary transport mechanism, enforcement of the angle of repose (through avalanching) and deposition sinks in the shelter of relief (‘shadow zones’). The model produces a range of dune types observed in nature, including barchans, transverse dunes, linear dunes and star dunes. Self-organization of these dune patterns results from the nonlinear dynamics between the local sand transport rates, the migration rates of the evolving heaps of sand and the shadow zones and avalanching mechanisms. Werner also proposed a description of the self-organization process in terms of attractors, quantified in phase-space by the number of dune crest terminations in the dune pattern and the dune orientation relative to the resultant (mean) sediment transport flux. A very similar model developed by Nishimori et al. (1998) is capable of simulating a range of dune types. Nishimori et al., however, define the dune type attractors in terms of wind directional variability and the amount of sand available in the system. Most recently, a modification of the Werner model by Momiji et al. (2000) incorporates the effects of wind speedup on the stoss slopes of developing dunes. This results in the evolution of a more realistic cross-sectional profile of the dunes with less steep windward slopes and it also introduces an equilibrium limit to the size of the dunes in the model space.

While these models provide an important tool for understanding the formation of different dune types and patterns in terms of self-organization, their significance to coastal geomorphology is limited because the critical element of vegetation is not included. The interactions between vegetation and sediment transport are decisive components in the dynamics of coastal dune landscapes, constituting an additional set of complex feedback processes. The presence of vegetation results in very different types of dune patterns such as hummocks, foredunes, parabolic dunes and blowouts, and it plays a crucial role even in semiarid environments (Hesp, 1989; Carter et al., 1990; Pye and Tsoar, 1990; Lancaster, 1995). The following section of this paper introduces a substantial extension of the Werner model, incorporating vegetation in the simulated landscape, to illustrate the viability of self-organization approaches to the reproduction (or simulation) of coastal landform systems. The modeling of vegetation dynamics of growth and decline as a result of burial and erosion and its effects on sediment transport allow for a more realistic simulation of coastal dune landscape development.

5. Numerical simulation model

The simulation model is based on the original algorithm of Werner (1995), also outlined in Momiji et al. (2000). The principal feature of the algorithm is that batches of sand are transported across a simulated 3D surface based on a stochastic procedure, whereby the erosion, transport and deposition processes are determined by chance. The model area consists of a square cellular grid containing stacked slabs of sand of a fixed height that constitute the topography. The sand transport process is simulated by moving consecutive slabs across the grid. The edges of the grid area are connected by periodic boundaries so that exiting slabs are brought back into the model area on the opposite side of the grid.

Simulation of the sand transport starts with a random selection of a grid cell as an erosion site and if that grid cell contains sand, the top slab is removed and taken up for transport. The base of the model area is considered to be a stratigraphic layer below which further erosion is not possible. After erosion, the slab is moved along a transport trajectory, L, toward a new position on the grid (see Fig. 3). This transport trajectory represents the movement of the sand by the wind. At the arrival site, deposition is determined by chance, affected by conditions at the

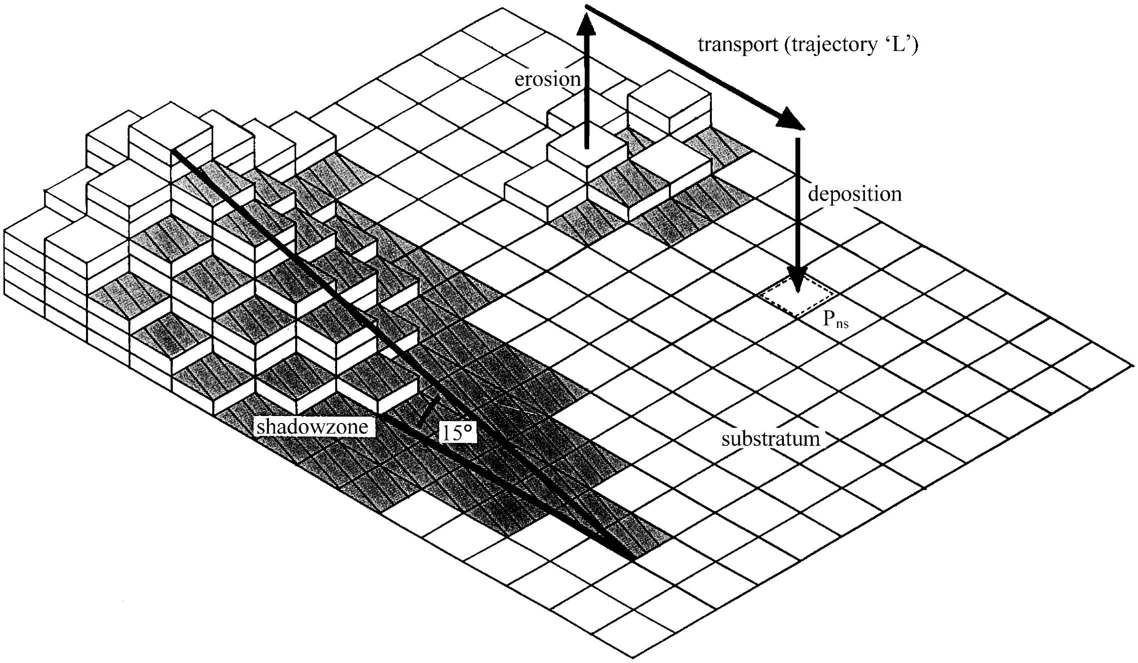

Fig. 3. Schematic representation of the slab-covered grid, sand transport process and the shadow zone in the simulation algorithm. Shaded cells are located in a shadow zone.

designated grid cell. If the slab is determined to be deposited, the stack of slabs at that grid cell is increased by one; if the slab is determined not to be deposited, the slab is moved one transport trajectory further and a new deposition assessment takes place. This procedure is repeated until the slab is deposited.

In this algorithm, only the erosion and deposition processes are stochastically controlled. The transport trajectory consists of a vector with x and y components that are parameters set at the beginning of the simulation, representing the force and direction of the wind. The erosion process is controlled by a probability of erosion at each grid cell. In the original algorithm, this probability is set to 1, i.e. once a cell is selected as an erosion site, a present slab is always removed and transported. The deposition process is controlled by a probability of deposition, p, based on whether the deposition cell already contains slabs of sand, p_s, or not, p_{ns} (i.e. when the underlying hardrock base is exposed).

Besides the main sand transport process, two constraints are simulated in the model: shadow zones and avalanching. A shadow zone is the area in the lee side of relief where wind flow has been slowed down sufficiently to suppress any further transport of sand. In the model, this is represented by a shelter zone downwind of relief covering the area enclosed by an angle of 15° (the shadow zone angle, β) to the horizontal from the top of the relief (see Fig. 3), in which deposition probability is 1 and erosion probability is 0. Avalanching is simulated to maintain the angle of repose of loosely packed sand, which is usually an angle of 33° to the horizontal. After removal or addition of a slab of sand (erosion or deposition), the model assesses whether this angle of repose is exceeded and if so, moves neighboring slabs down the steepest slope (in the case of erosion) to reinforce this angle or moves the newly deposited slab down the steepest slope until it reaches a grid cell where it does not violate the angle of repose. Avalanching is the only means of sideways sand transport relative to the transport trajectory.

Time evolution in the model is produced by repeating the slab movement process and is recorded by the number of iterations, where one iteration amounts to a quantity of consecutive slab transports equal to the amount of grid cells in the model area. This does not imply that every cell has been polled for erosion. Various cells can be chosen more than once during a single iteration, while other cells are skipped because of the random site selection. Since slab height is expressed as a ratio of the cell dimensions and iterations are based on grid size, both the spatial and temporal dimensions are undefined. As a result, the model can be coupled to reality by defining one of the dimensions and scaling others to the desired specifications.

In order to bring vegetation into the model environment, a number of alterations and extensions is introduced to the original algorithm. Each grid cell now contains two variables: (1) the number of sand slabs at that site (as originally mentioned) and (2) an additional variable describing the influence of vegetation at that site on the erosion and deposition processes. This variable, referred to as 'vegetation effectiveness', can be interpreted as a coverage density or a frontal area index (FAI) and has a value between 0 and 1 (or between 0% and 100%). This vegetation effectiveness affects the erosion and deposition process, but not the intermediate transport trajectory. It alters erosion probability at a cell in a linear relationship whereby 0% effectiveness results in an erosion probability of 1.0 (as in the original model) and 100% effectiveness decreases the erosion probability to 0.0 (i.e. no erosion possible). Deposition probability is affected in a similar linear manner, but starting from the original p_s or p_{ns}, where p rises to 1.0 at 100% vegetation effectiveness. This approach simulates the well-documented influences of vegetation on the threshold shear velocity required to initiate and sustain sand transport rates (Wasson and Nanninga, 1986; Raupach et al., 1993; Hagen and Armbrust, 1994; Lancaster and Baas, 1998). Although the exact functional relationship is complex and most likely not linear, the simplistic assumptions for simulating the vegetation influences described above are deliberately chosen to be consistent with the elementary modeling of the sand transport in the basic algorithm.

Vegetation is not a static fixture in the landscape. Considering species like marram grass (*Ammophila arenaria*) in coastal dune environments, the vegetation responds to fresh sand input and burial by growth and generally shows a more vital character (Disraeli, 1984; Fay and Jeffrey, 1992; De Rooij-Van der Goes et al., 1998; Maun, 1998). In the model, the development of the vegetation in the landscape is controlled by simplistic 'growth functions' that relate the erosion/deposition balance at each grid cell with the increase or decrease of vegetation effectiveness. This alteration of vegetation effectiveness is determined at the end of a vegetation cycle—defined as a number of iterations—introducing a periodic time scale in the model corresponding with the real-time yearly cycle.

The growth functions used in this model can be divided into: (1) dynamic vegetation with a positive feedback to sand deposition, e.g. marram grass (*A. arenaria*) and (2) conservative vegetation requiring a more stable environment with less extreme erosion or deposition, comparable to more shrubby vegetation (see Fig. 4). Though the botanical and agricultural literature contains many quantitative descriptions of vegetation response to burial and erosion, extensive qualitative data is much harder to compile, especially since a multitude of factors may further control the vegetation response under natural conditions, such as nutrient availability, soil fungi, light penetration to roots, salt spray, species competition, etc. (Van der Putten, 1993; Yuan et al., 1993; De Rooij-Van der Goes et al., 1995; Voesenek et al., 1998; Cheplick and Demetri, 1999). Furthermore, the subsequent vegetation response by means of changes in leaf area, plant height and coverage density cannot easily be related to its effects on the shear velocity threshold and sand transport rates. However, in order to remain consistent with the simplicity of the basic transport algorithm, the growth functions shown in Fig. 4 are deliberate simplifications that can only be described in terms of general characteristics, such as steepness (i.e. rate of response to burial or erosion) and the location of the peak (the optimum growth with respect to the erosion/deposition balance).

The vegetation cycle introduces in the algorithm a defined relation between the temporal scale (the yearly or seasonal periodicity) and the spatial scales (the erosion/deposition balance). As a result, transport trajectories and growth functions must be related to a specification of the cell size and slab height beforehand, and the scale invariance of the model is lost.

To augment the above description of the model algorithm, Fig. 5 depicts a flow scheme for one cycle

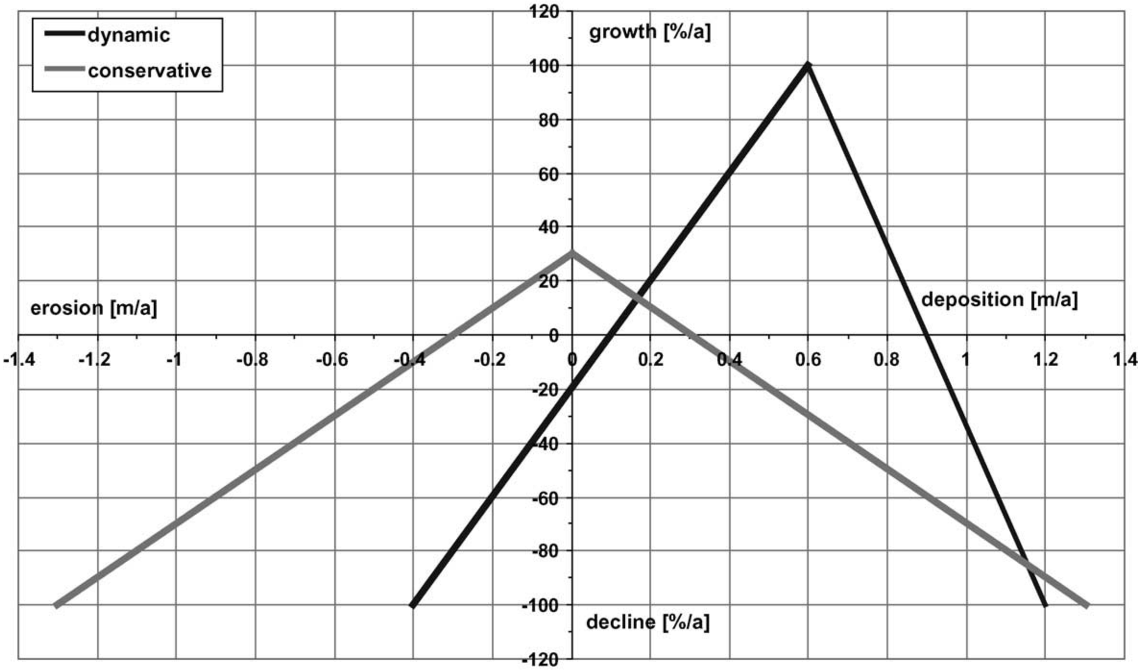

Fig. 4. Two different growth functions employed during simulations of vegetated dune landscapes. The black graph represents a marram-like vegetation with positive response to burial; the gray graph represents shrub-like vegetation with limited tolerance to erosion and deposition.

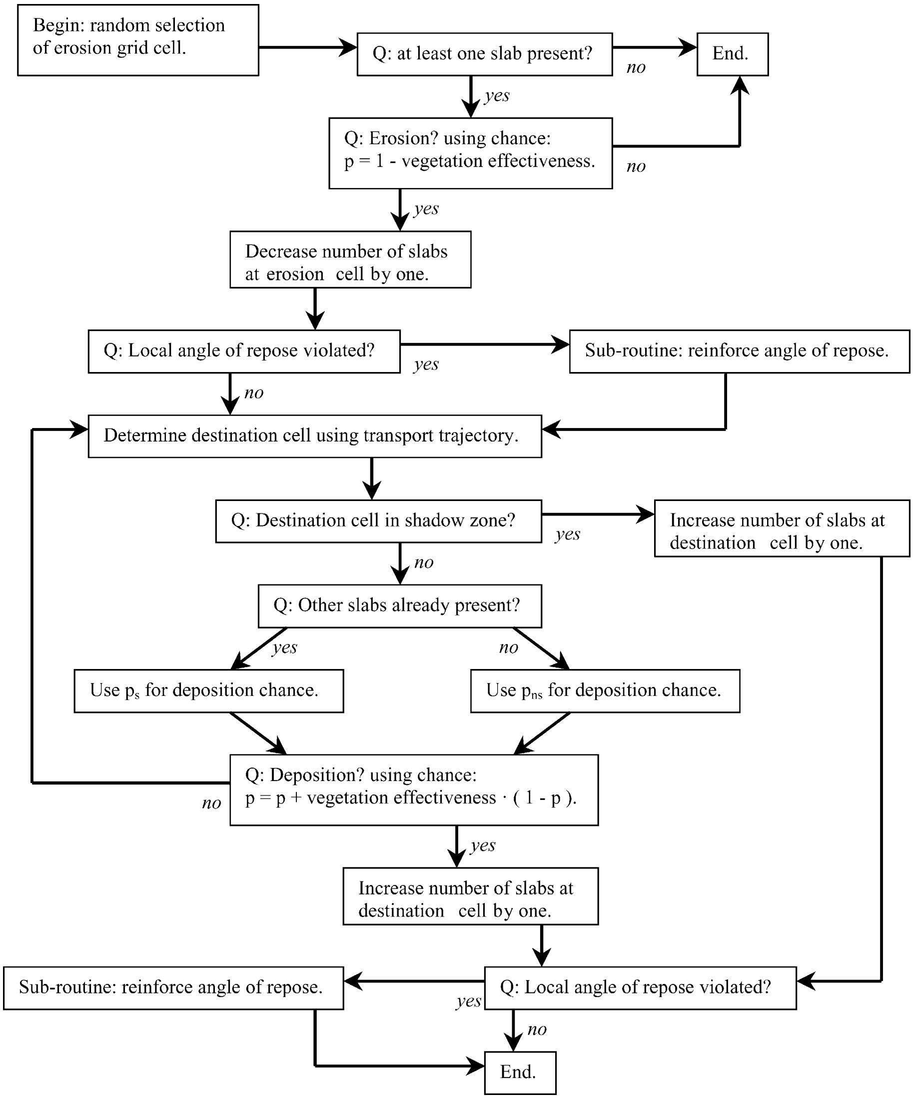

Fig. 5. Flow scheme for one erosion, transport and deposition cycle of an individual sand slab. Time evolution in the model is developed by repetition of this process. Vegetation effectiveness is adjusted after a set number of iterations, based on the accumulated erosion/deposition balance at each cell and the growth function employed.

of erosion, transport and deposition of an individual sand slab.

6. Results

Initial investigations were conducted to reproduce the types of dune landscapes previously simulated by Werner without the vegetation influences in the environment. Fig. 6 shows an example of a barchan dune field simulation, evolved from an initially random undulating topography with no vegetation present. Other bare sand dune types, such as transverse dunes, seif dunes and star dunes, were successfully modeled as well. Since this class of simulations has already been described by Werner (1995) and Momiji et al. (2000), this paper focuses on the modeling of dune landscapes in the presence of vegetation. For a full

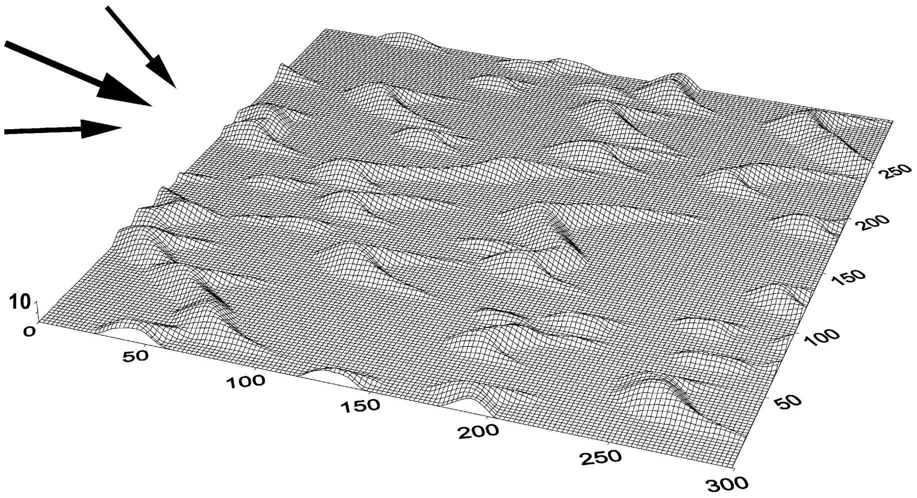

Fig. 6. Barchan dune field developed from initially flat, randomly undulated topography under unidirectional wind regime after 600 iterations. A sequence of three transport trajectories was used (indicated by arrows): the main trajectory is 5 cells long, blowing for 50% of the time; two oblique trajectories are 3.6 cells long at an angle of 33° with main trajectory, blowing for 25% of the time each. Grid dimensions: 300 × 300 cells.

description of the simulations of bare sand dune landscapes (such as transverse dunes and star dunes) the reader is referred to the above articles.

During simulations with vegetation, the following default parameter settings were used: angle of repose = 30°; angle of shadow zone (β) = 15°; slab height = 0.1; $p_s = 0.6$; $p_{ns} = 0.4$. The difference between p_s and p_{ns} conveys the better saltation rebound on hardrock as opposed to a sand surface (Bagnold, 1941). Multidirectional wind regimes are simulated using a cyclic series of differing transport trajectories. These parameter settings agree with the ones employed by both Werner (1995) and Momiji et al. (2000) in their simulations. The growth functions are evaluated at periods of 12 iterations, where one iteration represents 1 month, and the evaluation event roughly corresponds to the growth season (though the algorithm does not capture true seasonality). Simulating a year per cycle, the cell dimensions are set to 1 m (square) and subsequently, the standard slab height to 10 cm. The two principal growth functions employed are described by two characteristics (see Fig. 4): (1) the maximum of the function, defined by the optimal deposition/erosion rate (x-coordinate) where the optimal growth occurs (y-value) and (2) the steepness of the function defining the response rate of the vegetation species: the steeper the function, the faster the vegetation responds to changes in the erosion/deposition balance. The dynamic growth function corresponding to marram grass has its maximum at 0.6 m of deposition per vegetation cycle, while the vegetation effectiveness declines when deposition rates fall below 0.1 m/year. The conservative growth function in contrast reaches its optimum growth at a zero balance and declines when either erosion or deposition rates exceed 0.3 m/year.

In order to vary sand transport rates in the model, the slab height, rather than the transport trajectories, are adjusted using values of 0.2, 0.1, 0.05 to 0.02 m, representing high to low transport conditions. As the length of the transport trajectory was usually set to 5 m, this results in a transport rate of 0.8 m^3/m/month for a slab height of 0.1 m, a realistic figure for moderate sand transport conditions in dune landscapes (Bagnold, 1941; Goldsmith et al., 1990; Sherman and Hotta, 1990; Arens, 1994). The simulations are initiated with a flat layer of sand overlying a hardrock substratum covered with optimal vegetation (effectiveness 100%). A circular patch of bare sand, representing a break in the vegetation, is situated near the upwind border of the model area.

Fig. 7 shows the development of a dune landscape containing a dynamic vegetation under high sand transport conditions (slab height = 0.1) in one direction (left to right). The gray scale in the figure shows the varying vegetation effectiveness throughout the

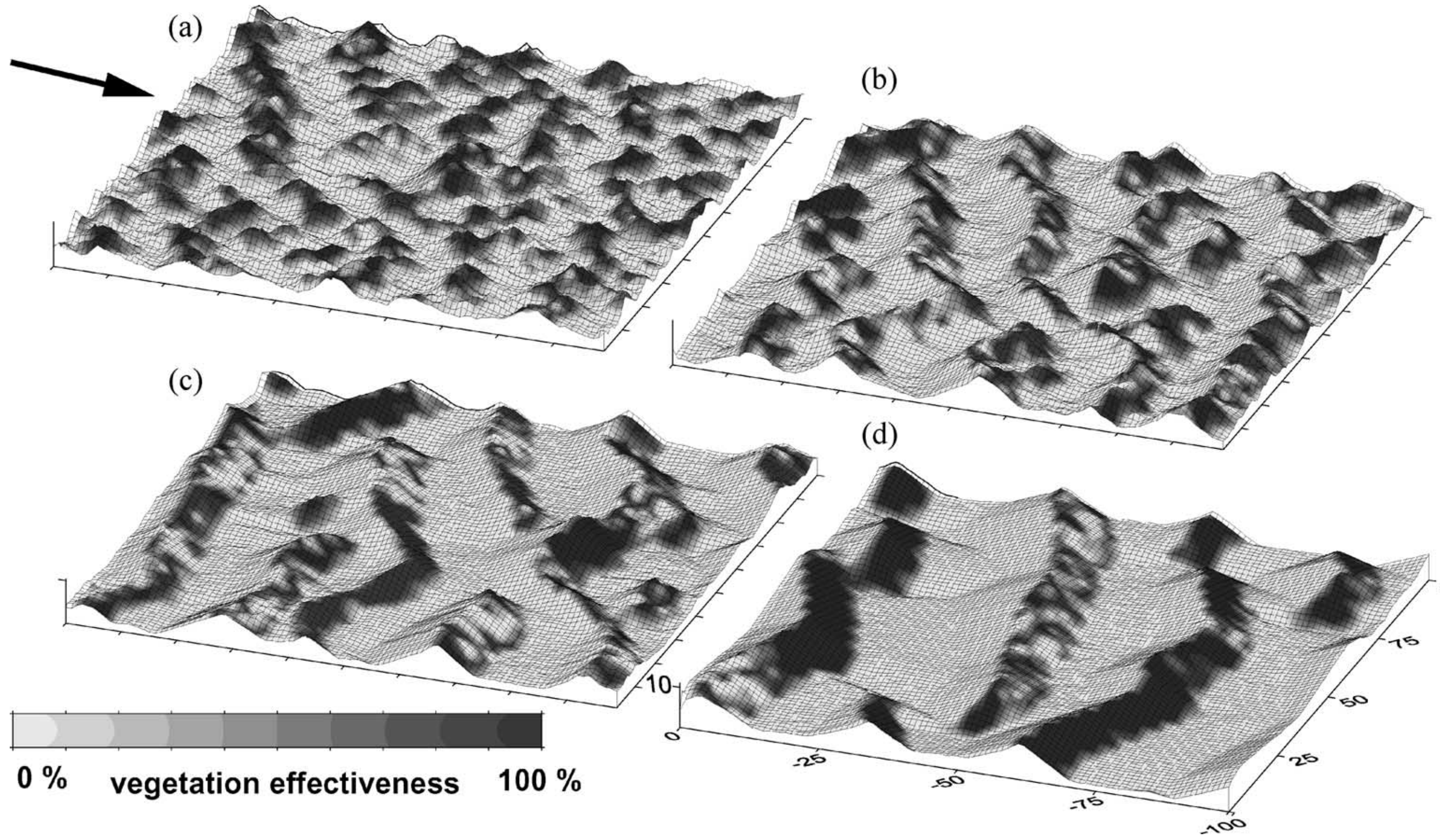

Fig. 7. (a–d) Development of a vegetated landscape out of an initial flat surface with one bare patch in the center. High transport conditions (slab height = 0.1, length = 5), from left to right (direction indicated by arrow) and a dynamic vegetation. Height and distance in meters. The diagram shows the landscape after 5, 10, 20 and 50 years in (a), (b), (c) and (d), respectively. Grid dimensions: 100 × 100 cells.

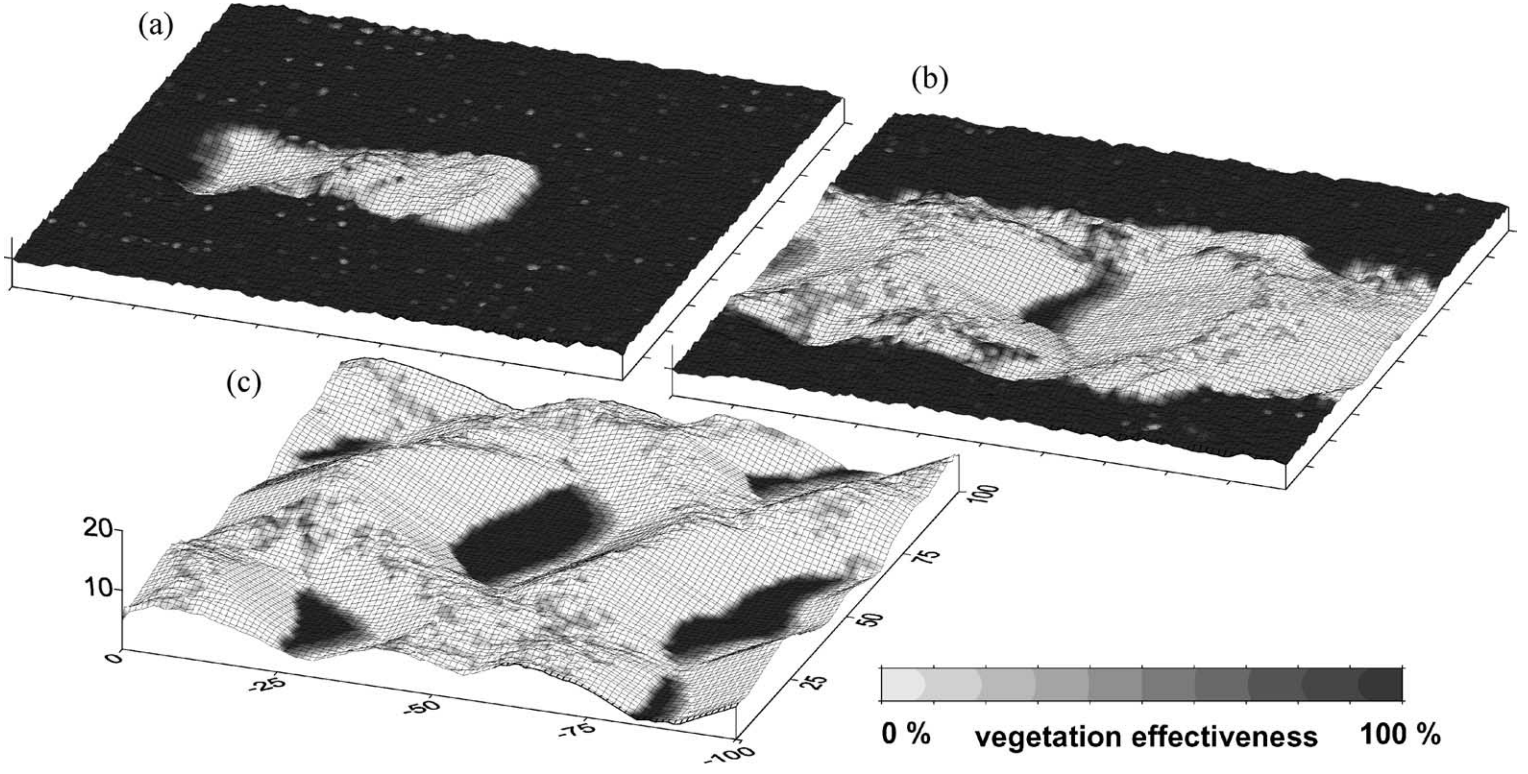

Fig. 8. (a–c) Development of a depositional lobe and sideways expansion out of an initial flat surface with a bare patch at the upwind border. High transport conditions (slab height = 0.1, length = 5), from left to right (direction indicated by arrow) and a conservative vegetation. Height and distance in meters. From top-left to bottom shows landscape after 10, 20 and 50 years in (a), (b) and (c), respectively. Grid dimensions: 100 × 100 cells.

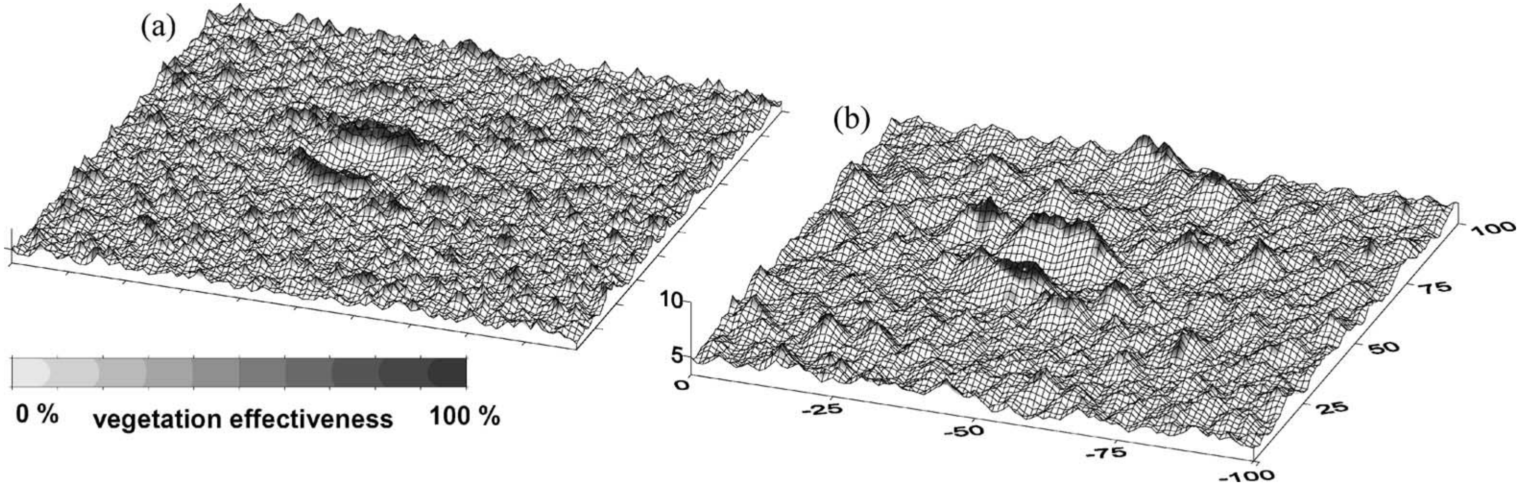

Fig. 9. (a, b) Landscape development out of an initial flat surface with a bare patch in the center, using 12 different transport trajectories within one vegetation cycle. These 12 trajectories attempt to simulate the sand transport regime in Dutch coastal dunes, with trajectories from various directions and various lengths. The used growth function is the dynamic deposition-dependent vegetation. The top of the model area is north; major transport trajectories are from southwest, west, northwest and northeast; secondary trajectories are from east, south and west. Notice that the height scale is exaggerated, relative to Figs. 4 and 5. From left to right shows landscape after 10 and 20 years in (a) and (b), respectively. Grid dimensions: 100 × 100 cells.

landscape. The initial vegetated surface is quickly activated throughout the model area and a complex landscape develops, evolving from a hummocky topography through a stage with crescentic ridges towards more or less transverse ridges. While vegetation occupies the tops of the hummocks in the early stages, it is mainly limited to the slip faces in the final stage and does not appear to have any role in fixing the sides of the developing dunes, the process controlling parabolic dune development (Greeley and Iversen, 1985, p. 173; Carter et al., 1990; Pye and Tsoar, 1990, p. 201). The dynamic vegetation is not capable of confining the dune development to the initial bare patch; instead, the whole model area rapidly changes into a dune landscape. Other simulations using varying sand transport rates and growth functions with small variations in tolerance produce the same type of dune landscape development, with only minor differences.

Using a conservative vegetation and high sand transport conditions, a more parabolic shaped and confined dune develops initially, shown in Fig. 8a, under a unidirectional wind regime. Simultaneous invasion of the upwind border of the bare patch by the vegetation is also clearly visible. However, as the development progresses and the main dune structure migrates through the model area, the sideways expansion of the dune is not restricted by the vegetation and true trailing ridges do not develop. Instead, the dune development extends throughout the model area and eventually, the vegetation is restricted to the interdune valleys (Fig. 8b and c).

Finally, a multidirectional wind regime was simulated, representing a sequence of monthly average sand transport rates and directions in the Dutch coastal dunes throughout the year, with a dynamic vegetation (Fig. 9; initial bare patch was situated in the middle of the grid). This simulation shows striking differences with the other two described above in that: (1) the area surrounding the original bare patch develops strongly while the rest of the model area initially remains relatively flat, (2) the two main dune bodies develop parallel to the mostly gentle prevailing westerly winds (from left to right) and transverse to the oblique or normal storm winds (northerly and southerly winds) and (3) the vegetation is generally confined to the tops of the dunes instead of in the troughs or on the slip faces.

7. Discussion

The above modeling efforts have been merely exploratory, but they have produced some tantalizing results. They clearly show the potential of this approach for simulating strikingly different and realistic dune patterns under the influence of vegetation dynamics. The great contrast in appearance between the landscapes of Figs. 7 and 8, for example, shows the large impact of vegetation dynamics on the developing morphology. It illustrates how a change in parameters (here the growth functions) results in a fundamentally different landscape. The development of the multidirectional wind regime simulation (Fig. 9) is not as straightforward to interpret. After 20 model years (Fig. 9b), the area surrounding the original bare patch seems to develop into a hummocky landscape, where the hummocks are anchored at their tops by the vegetation. This type of landscape, however, would not be expected with a deposition-dependent vegetation. Further simulations are required to investigate thoroughly the sensitive dependence of the resulting landscapes on the various modeling parameters, most notably, sand transport conditions and vegetation response. Such efforts may reveal certain attractor landscapes or, alternatively, a chaotic behavior where small changes in the parameters result in drastically different morphology and vegetation patterns.

In terms of self-organization, the driving force in the system is the transport of sand by wind—externally supplied energy—which creates elevation differences and hence, potential energy in the landscape when sand collects in dune morphology. Chaotic behavior is exhibited by the movement of individual slabs of sand: their trajectories are unpredictable due to the complex interactions between the erosion and deposition rules and the morphology and vegetation in the landscape. The self-organizing mechanism of dune formation is the inverse proportionality between the migration rate of heaps of sand and their size, which allows smaller heaps of sand to catch up with larger ones and to merge into dunes. The created potential energy is dissipated through the process of avalanching which turns the potential energy into kinetic energy and finally into frictional heat after coming to rest at a lower elevation. In this context, the dunes are considered dissipative structures in a far-from-equilibrium situation (the equilibrium situation would be a flat stable plane with erosion and deposition balancing in each cell). The incorporation of vegetation introduces additional dynamic feedback loops as vegetation effectiveness modifies sand transport and vice versa. It induces a second self-organizing mechanism in the form of positive and negative feedback between deposition and erosion on the one hand and vegetation effectiveness on the other.

Although the self-organization aspects of the model algorithm are broadly understood, the interpretation of the resulting landscapes in terms of attractors is not as clear. In Werner's original model, the attractors are defined by the number of dune crest terminations and the orientation of the crests to the mean transport direction in the landscape. This approach differentiates barchans, transverse dunes and linear dunes, but does not identify seif dunes and star dune patterns quite as well. A more traditional evaluation in terms of the wind directional variability (RDP/DP ratio) and the equivalent sand thickness (Wasson and Hyde, 1983) could provide a better attractor quantification in this respect. This still does not capture the vegetation patterns and effects in the coastal dune simulations, however, and is not very suitable for differentiating these vegetated landscapes. At present, there exists no quantitative evaluation for categorizing vegetated (coastal) dune landscapes, most likely due to the fact that it is difficult to quantify a vegetation pattern together with its spatial correlation with the morphology.

The simulation algorithm requires improvement in several areas. First, the joint presence of more than one vegetation species in a grid cell must be accommodated, so that both dynamic and conservative growth functions interact with the transport dynamics simultaneously. In the last simulation effort described above (for the Dutch coastal dunes, Fig. 9), for example, the relatively stable flat areas farther from the original bare patch would have been occupied by conservative vegetation if both dynamic and conservative growth functions had been jointly present in the modeling environment. As a result, a more realistic mosaic of vegetation patterns and morphology would likely have been achieved. Second, the modification of the Werner model proposed by Momiji et al. (2000) in terms of a wind speed-up factor on stoss slopes will provide for a more accurate development of the cross-sectional profile of the dune morphology. The other important future objective should be the development of appropriate phase-space variables for the quantification of possible attractor landscapes that incorporate features of both morphology and vegetation.

Many further modifications are easily devised, such as more complicated growth functions, interactions between the trajectory and the intermediate cells, varying shadow zones and so forth. Such refinements, however, are not expected to significantly change the fundamental characteristics of the self-organized dune landscapes. More importantly, the strength of this model and of every self-organization model is the selection of only the most fundamental processes and interactions acting in the landscape and simulating these in the most simplistic manner. Attempts to incorporate more detailed features of aerodynamics and sand transport would obscure the fundamental self-organization operations, introduce a myriad of adjustable parameters and confound ultimate attractor interpretation.

The implications of these exploratory modeling efforts are of a mostly general nature. First, the success of this model in generating relatively realistic coastal dune landscapes demonstrates that it is certainly feasible to identify a specific set of fundamental processes that capture the full dynamics on a landscape-scale level. Second, these fundamental processes need not incorporate all the intricate details present on a smaller scale. Secondary wind flow patterns induced by relief, for example, are apparently not essential to the development of a realistic dune topography. Third, this type of approach provides a connection between descriptions of a more geologic or physiographic nature and small-scale deterministic processes. For example, the model simulates the interaction between a large-scale wind climatology and the small-scale process of sand transport. Finally, the implementation of self-organization concepts can raise new or extended research questions. In this case, for example, the issue of quantitative knowledge on vegetation response to burial or erosion of sand takes on new prominence.

8. Conclusion

Geomorphological research has been greatly enriched by the concepts of chaos, fractals and self-organization developed over the past few decades. Since nearly all geomorphic systems involve complex nonlinear dynamics, they are inherently amenable to these types of investigations. In coastal geomorphology, this has inspired research ranging from wave climates to shore profiles and beach cusps. The description of a system in terms of self-organization and attractors provides for an alternative analysis powerful for its simplicity and transparency, considering only those processes fundamental to the system's evolution and excluding minor reductionist minutiae. The potential of this approach is readily seen in the dune simulations by Werner (1995), Nishimori et al. (1998) and Momiji et al. (2000) that are able to produce a range of recognized 3D dune patterns without simulating complex aerodynamics and sand transport processes. The introduction of vegetation in the simulation model—as presented here—is able to capture an even wider variety of dune landscapes, with only a minimum of interactions added to the algorithm. Interpretation of these tantalizing results, however, is presently frustrated by the lack of suitable quantitative attractor descriptions and phase variables that include the vegetation element in the landscape.

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